Using the Poisson Probability Distribution to Estimate Cost of Re-supply Spares
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An important aspect of the operational cost of a piece of equipment is the cost of spares. Failures of components of the equipment that are subject only to random failures (as opposed to wear-out, infant mortality, and other non-random failure modes) can be modeled with the Poisson distribution. If the equipment can be continuously re-supplied, the number of failures of such components over an operational period totaling H hours is equal H / MTBF where MTBF is the mean time between failures. If each spare has a cost C then the cost of the spares over H hours is C*H/MTBF.

The situation changes when re-supply is not continuous. If the equipment is isolated from its source of supply for a substantial time, as happens at remote storage or operational locations and on certain mobile systems such as ships, then to keep the equipment operational spares must be carried along with or kept "near" the equipment. An important issue is how many spares to stock? If the spares run out before the next re-supply then the equipment will become unavailable. This could have serious consequences.

Theoretically, for perfect assurance there will be enough spares the number of spares must be unlimited, but this implies unlimited cost and unlimited spares storage space. In practice, the number of spares for any component must be limited. In such situations, there is a tradeoff between cost (and perhaps available space and other factors) and the statistical *confidence* there will be enough spares. To keep the analysis simple, we consider only the cost tradeoff.

The appropriate probability distribution for the number of failures of a component per unit of time is the Poisson. A random variable X (e.g., the number of failures in a given period of time, say an hour) is said to have the Poisson distribution if

$$P(x ; \lambda) = (e^{-\lambda} \lambda^{x})/x! \quad x = 0, 1, 2, \ldots$$

for some $\lambda > 0$.

Here λ is the average failure rate, e.g., in failures per hour.

Example 1: Let us calculate the probability of exactly zero failures in 5,000 hours of operation if the average failure rate is once every 5,000 hours.

$$P(0 ; 1) = e^{-1}\ 1^{-0})0! = 0.3679$$

Example 2: As in Example 1, except calculate the probability of exactly one failure.

$$P(1 ; 1) = e^{-1} 1^{-1})1! = 0.3679$$

Example 3: As in Example 1, except calculate the probability of exactly two failures.

$$P(2 ; 1) = e^{-1} 1^{-2})2! = 0.1839$$

Example 4: As in Example 1, except calculate the probability of exactly three failures.

$$P(3 ; 1) = e^{-1} 1^{-3})3! = 0.0613$$

Example 5: In the above examples, what is the probability of three or fewer failures? Sum the probabilities in the above four examples to get 0.9197.

Example 6: A ship leaves port with its radar operational. It will not be able to re-supply for 5,000 hours. The magnetron is an essential component of the radar. Its average failure rate is once per 5,000 hours. In addition to the magnetron already in the radar, the ship carries two spares. What is the probability that all three magnetrons will fail and that the ship will therefore be without an operational radar? The probability of all three magnetrons failing was given in Example 4. The probability of not having an operational radar is therefore 0.0613. Conversely, the probability of having an operational radar for the entire voyage is 1 – 0.0613 = 0.9387.

Example 7: Suppose that the insurers of the ship in the previous example insist that the ship carry enough spare magnetrons to assure a 98% probability of not running out. How many magnetrons must it carry for a 5,000 hour trip?

We proceed by trial and error. We have already learned that with three magnetrons the probability of all three failing is 0.9387. If we add a fourth magnetron, we calculate the probability of exactly four failures as

$$P(4 ; 1) = e^{-1} 1^{-4})4! = 0.0153$$